FEEDBACK IN PATH ANALYSIS: SOME SOCIOLOGICAL EXAMPLES

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I. INTRODUCTION

There has been a considerable amount of work incorporating path analysis in the sociological methodology since Boudon's (1965) and Duncan's (1966) introductions of dependence or path analysis to social science research, which is based on Wright's (1921, 1934) pioneering works. Sociologists and other social scientists have found this type of analysis to be a useful tool in attempting to bridge the gap between sociological theory and statistical analysis. The linear causal models have gained a greater prominence due to path analysis.

With few exceptions path analysis has been used only with unidirectional causal models. Although statistical principles for the consideration of reciprocal interaction (feedback) in path analysis are available (e.g., Turner and Stevens, 1959; Tukey, 1954; Wright, 1960b) they have been largely ignored by sociologists who have used path analysis. Duncan (1966) noted that feedback in path analysis was a neglected aspect of the technique. With only one exception (Duncan, Haller and Portes, 1968; Duncan, 1970) our literature review of sociological research which has used path analysis indicates that feedback remains a neglected topic.

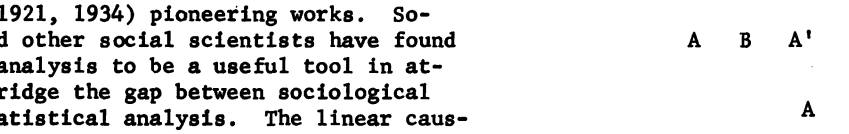
This benign neglect can be attributed to several factors: (1) the system of reciprocal interaction is not a topic in vogue yet although it is talked about in sociological research, (2) from a methodological perspective, reciprocity or feedback cannot be subjected to the simple interpretation as is the case for unidirectional causal models (3) since sociological research tends to deal more with cross-sectional data rather than longitudinal data, the need for grappling with the concept of feedback may not yet have been seriously felt, (4) sociologists may not yet recognize reciprocal interaction or feedback as a useful theoretical and methodological device. Other factors that might help explain why feedback has been neglected could be presented, but that is not the purpose of this paper. Regardless of the reasons, feedback largely has been ignored by sociologists who have used the path analytic technique.

The major concern of this paper is to discuss the issue of feedback in path analysis from a sociological research perspective. It should be noted that such a model will require somewhat different theoretical, statistical, causal, measurement and sampling assumptions (discussed in Section III) than with the unidirectional path analysis. We also will provide theoretical and empirical examples of feedback models in sociological research, and we will examine how feedback can be handled from the path analysis perspective.

II. THE CONCEPT OF FEEDBACK AS A METHODOLOGICAL DEVICE

In its simplest form, feedback or reciprocal interaction may be defined as a continuous "cause and effect" sequence in a system of relationships. To illustrate, variable A affects variable B with some defined function, and variable B also affects variable A with some defined function. A pictorial representation of this type of feedback can be shown as:

FIGURE 1



What is important here is that the variables are in some way interacting. There are two basic types of feedback that may be considered: (1) instantaneous feed (feedback with time lag) and (2) delayed feedback (feedback with time lag). These distinctions are made on the basis of whether time is a real parameter in the reciprocal interaction of two or more variables. For example, the reciprocal interaction relationship A B can be thought of as A_t B A_t is a measurement of variable A at time one and A_t is a measurement of the same variable at time two. In other words, there is a time lag between the two measurements of A.

An advantage of the feedback is that it does not require all of the assumptions of causal models. The utility of feedback model is maximized if we keep in mind that usually the research questions raised are not always subject to unidirectional causal models. Frequently, we are unable to meet some of the causal assumptions or establish the causal priorities necessary for unidirectional causal models, and our research efforts become fraught with serious theoretical and methodological problems.

Much of the sociological research concerned with temporal dimensions involves the reciprocal interaction or feedback situations. For example, two interrelated propositions from demographic research state that (1) as the fertility rates increase, the infant mortality rates also increase, (2) as the infant mortality rates decrease, the fertility rates also decrease (Bogue, 1969: 831). Another example can be drawn from the research on modernization and urbanization. The two propositions can be stated as: (1) the greater the modernization of a society, the greater the urbanization, and (2) the greater the urbanization of a society, the greater the modernization (Lerner, 1958; Black, 1967). At first glance these examples might appear to be circular. A closer examination will indicate that there are reciprocal relationships between (1) fertility rates and infant mortality rates, and (2) modernization and urbanization of societies. Similar examples can be drawn from many social/psychological and sociological research.

In our efforts to make sociology consistent

with the goals of science, we attempt to arrive at (1) estimation, and (2) structural models (Heise, 1969). While the current perspective of path analysis in sociology is sufficient to deal with estimation models, it is not adequate for analyzing structural models. It is our contention that sociological theories will be more accurately tested through the use of structural models than through the use of estimation models.

III. ASSUMPTIONS[1]

The reciprocal interaction (feedback) in path analysis may either include or ignore any time lags. While the assumptions discussed below usually apply to both of these situations.

1. There is a reciprocal relationship between the variables under consideration.

This assumption suggest that if A and B are reciprocally related with each other, either A or B can be used as a starting point for analysis. While it may be feasible (or even sometime pragmatic) to establish a causal order, it is essential that both A and B act in a "cause and effect" sequence. The notion of recursive systems (especially causal order) may be helpful in determining a starting point for the analysis (Blalock, 1964, 1967).

2. The variables have linear relationships.

This assumption is frequently made but usually not tested in sociological research. If known nonlinear relationships do exist, they can be reduced to linear form by transformations. If transformation is necessary, the user should be aware of the difficulty of specifying parameters using this method.

3. The reciprocal interaction relationships of the system under study are sufficiently established from a theoretical point of view.

This assumption is often made in sociological literature but is rarely empirically established. Much of the research in sociology has been concerned with single variable situations, and, consequently, they did not establish interactive relationships. It should be pointed out that there are many "grand" and "middle range" social theories that are best tested with feedback models. At the same time, it can be readily argued that some of the "micro" theories may be more clearly subject to interactive systems of relationships than unidirectional causations. While it must be admitted that sociological theories are not developed enough to effectively utilize the path analytic and causal inference approaches, feedback models of path analysis may circumvent some of these problems. In addition, we tend to view human behavior, as well as human societies, with sets of interrelated (oftentimes circular) propositions. While we cannot debate, at this point, whether a unidirectional model or a feedback model better fits such theorizing, the writers submit that the latter might be a better approach.

4. Observations (measurement) of each variable are made to allow the intervals of maximum effects.

This assumption can be problematic in sociological research. The reciprocity of interactions can be thought of as a standardized normal distribution and, consequently, will reach a peak. The usual path analytic approach applies (in a strict sense) to point variables only. To achieve the intervals of maximum effects, Wright (1960: 424) suggested, "The intervals between observations or the lenghts of the averaged periods should be equal to or be an aliquot part of the lag." While such controls may be applied in studies of feedback with time lag, the problem may be somewhat less cumbersome in the instantaneous feedback situation. In the latter case, one can simply assume that these adjustments are being made both ways, and, consequently, the observations remain essentially linear point variables.

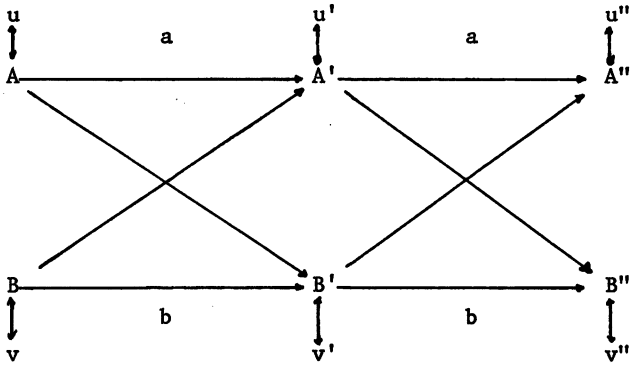
IV. FEEDBACK MODELS IN PATH ANALYSIS

As mentioned earlier, the mathematical formulation of feedback already is available (Wright, 1960; Duncan, Haller and Portes, 1968; Duncan, 1970). Other works in the area of econometrics, which have considered simultaneous structural equations include Johnston (1963), Goldberger (1964), Ezekiel and Fox (1959). Our focus in this section will be the application of feedback models of path analysis in sociological research. As previously noted, there are very few studies which have utilized such a perspective. Consequently, most of our examples will be theoretical in nature with an attempt to demonstrate how they might be used from reciprocal interaction perspectives. The examples to be discussed are bivariate, multivariate; and multivariate-multiphasic feedback models.

Bivariate Feedback Models

The simplest form of feedback model is the bivariate model as illustrated in Figure 2,

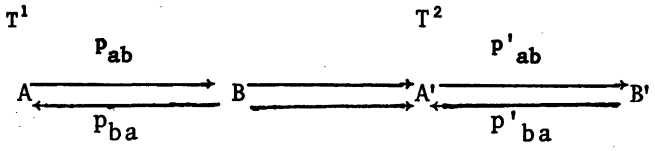
FIGURE 2



where A and B are the known variables and u and v are unknown residual variables. The above figure is an example of a path diagram illustrating delayed feedbacks between variables A and B and can

be extended through as many cycles as are necessary (Wright, 1960b). Another way of looking at this type of feedback model is from a multiphasic perspective (with known time parameters), which can be represented as shown in Figure 3.

FIGURE 3



where T = time and p = path coefficient.

The models of instantaneous feedback are perhaps more frequent in cross sectional research or these situations which imply known exogenous variables, and known interacting variables. An example of such a feedback model is given by Wright (1960b) for studying quantity marketed and price under the classical economic theory of supply and demand. Some sociological and social/psychological theories that can fit this type of model are exchange, interpersonal and human communication theories.

An example: A widely accepted argument proposed by Lerner (1958) is that urbanization and modernization are reciprocally related. In this proposition there are two unknown residual variables (A and B) that leads to the following:

$$\text{Urbanization (U)} = C_{um}\ \text{Modernization (M)} + C_{uaA} \qquad (2)$$

$$\text{Modernization (M)} = C_{mu}\ \text{Urbanization (U)} + C_{ubB} \qquad (3)$$

These equations may be stated in a standardized form as:

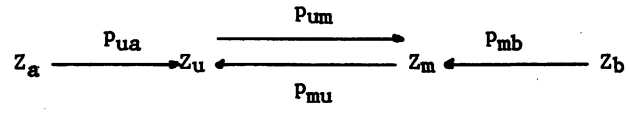
$$U = P_{um}Z_m + P_{ua}Z_a \qquad (4)$$

$$M = P_{mu}Z_u + P_{ub}Z_b \qquad (5)$$

Although we would rarely attempt to explain a bivariate model of reciprocal interactions, it will be of some value to understand its fundamentals. First, in this bivariate feedback model we find simple ways of looking at the reciprocal interaction without time lag. The equations may be extended with known time lags if there is need for it. The bivariate models can be easily extended into bivariate multiphasic models. It should be kept in mind that successive determinations will make such a model partially recursive. Second, as is true for the unidirectional path analysis, the reciprocal path coefficients will have simpler interpretation in the bivariate case than either the bivariate multiphasic or the multivariate multiphasic case (Land, 1969: 10-12).

The presentation of a bivariable model of this type is illustrated in Figure 4.

FIGURE 4

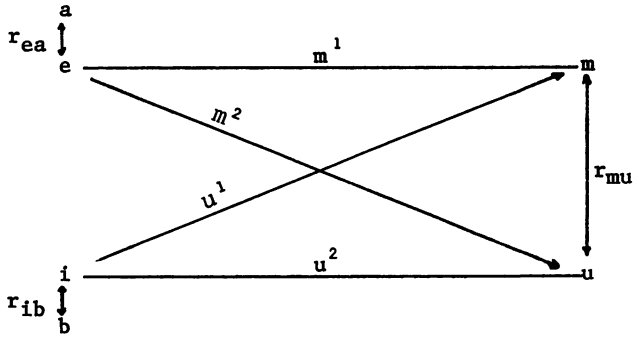


The computation of bivariate feedback path models will be quite similar to those for the simple bivariate path models with the exception that both variables will change ordinates as the case may be. There also will be two path coefficients and two coefficients of alienation.

Multivariate Feedback Path Models

In order to save space, readers are referred to Duncan, Haller and Portes (1968) and Duncan (1970) for specific computational details. Here we will attempt to expand the example suggested for the bivariate analysis. In addition to reciprocal interactions between urbanization and modernization, two known exogenous variables to this system are education and industrialization, as suggested by Lerner and others (e.g., Moore, 1963; Roger, 1969; Black, 1967). It may be assumed that the education and the industrialization of a society vary independently. Two unknown variables that are related with education and industrialization are A and B respectively. This model is represented in Figure 5.

FIGURE 5



where e = education, i = industrialization, m = modernization, u = urbanization, A and B = residual variables and r = correlation coefficient.

Considering the residual variable (A) we may write the following equations:

$$r_{ma} = m_1 r_{ea} \qquad (4)$$

$$r_{ua} = u_1 r_{ea} \qquad (5)$$

$$r_{mu} = m_1 u_1 + m_2 u_2 \qquad (6)$$

$$r_{mm} = m_1^2 + u_2^2 = 1 \qquad (7)$$

$$r_{uu} = u_1^2 + u_2^2 = 1 \qquad (8)$$

$$m_2^2 u_2^2 = (1 - m_1^2)(1 - u_1^2) \qquad (9)$$

$$(1 - m_2^2)_2(1 - u_2^2 u_1^2) = 1 - m_1 - u_1^2 - m_1^2 u_1^2 \qquad (10)$$

$$m_2^2 u_2^2 = (r_{mu} - m_1 u_1)^{'2} \quad (11)$$

$$(r_{mu} - m_1 u_1) = r_{mu} - 2m_1 u_1 r_{mu} + m_1 u_1 \quad (12)$$

$$m_1 = \frac{r_{ea}}{r_{ua}} u_1 \quad (13)$$

$$m = \sqrt{(1 - r_{mu}^2)/(1 - 2\frac{r_{ma}}{r_{mu}} + \frac{r_{ma}^2}{r_{ua}^2})} \quad (14)$$

$$u_2 = \sqrt{1 - u_1^2} \quad (15)$$

$$m_2 = \sqrt{1 - m_1^2} \quad (16)$$

Similar equations can be written for B if it is a known variable. The regression equations for m and u can be written as:

$$m - \bar{m} = C_{me} (e - \bar{u}) + C_{ui} (i - \bar{u}) \quad (17)$$

$$u - \bar{u} = C_{ue} (e - \bar{u}) + C_{mi} (i - \bar{u}) \quad (18)$$

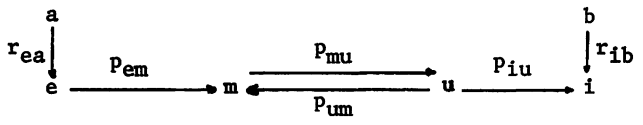
These coefficients can be calculated as ratios in which the unknown standard deviations cancel:

$$\varepsilon = \frac{u_1 \sigma_u}{m_1 \sigma_m} = C_{um} \quad (19)$$

$$\eta = \frac{u_2 \sigma_u}{m \; m} = \frac{1}{C_{um}}$$

Thus to illustrate the feedback between urbanization and modernization, our two endogenous variables, we can look at Figure 6.

FIGURE 6



The major consideration should be given to overidentification of structural equations. Such a problem can be approached from the "two stage least squares" (2SLS) perspective. Some suggestions have been made that reciprocal interaction equations can be expressed with "unilateral causal dependence" with positioned dependent variables and solved separately through least squares (Wold and Jureen, 1953). Such a set also can be thought of as a "recursive system" of relationship when delayed feedback models are used (Ezekiel and Fox, 1959: 927-28).

The reader is reminded of the unique problems of selections and measurement of variables in the preceding situations. Although some of the inclusions of "unnecessary" variables can be controlled through 2SLS, measurement may still be a problem.

Multivariate, Multiphasic Feedback Models

Such feedback models also may be considered as partially recursive in the sense that the endogenous variables are successively determined with a known causal time order and with a continuance "cause-effect" sequence. Such a model is discussed in Duncan, Haller and Portes' (1968) article.

This model can be delineated in two, three or four phases, as the case may be. One major problem, however, is the usual overdetermination of the model. Consequently, we will have more structural coefficients. This problem can be handled, of course, through multistage least square method, which we are suggesting as an extension of the two stage least square method. Such a method is suggested by Turner and Stevens (1959) and elaborated by Duncan, Haller and Portes (1968). Essentially, it implies a combination of causal modeling and factor analysis based on heuristic considerations.

V. METHODOLOGY OF FEEDBACK AND SOCIOLOGICAL THEORIES

In this paper we have alluded to the fact that our current sociological perspectives and feedback models and their interrelationships.

It should be obvious from the preceding discussion that feedback methods have considerable promise in theory construction and verification. While much focus in sociology has been upon construction of unidirectional causal theories, we need to keep in mind that we may be ignoring the understanding of the structure of sociological theories. While we do not deny the useful contribution of the unidirectional causal approach, in our opinions, the gap between sociological theories and research will be filled by utilizing reciprocal interaction models.

We can assume that in a crude way the major sociological traditions can be categorized into (1) evolutionary, (2) structural-functional, (3) conflict, and (4) cyclical perspectives. We can further assume that all of these traditions attempt (even though they may not succeed) to offer multilinear models of society. Thus, it becomes imperative that these theoretical traditions and derivations thereafter involve interdependence and often-times reciprocal relationships with or without time lag. In addition, explanations and dynamics of social change (which seem to be one of our major concerns without much directed effort) are embedded with sets of reciprocal interactions.

While we accept the fact that unidirectional causal models are first steps in constructing causal models, we need to extend the understanding of reciprocal relationships as the necessary second step for theory construction.

VI. CONCLUSIONS AND IMPLICATIONS

We are usually at a loss to utilize our most frequently discussed social theories because we have not had methodological tools to handle some theories. While the systems analysis approach has been available to us for some time, it has been largely unused due to needed assumptions of total

interdependence. We know that the variables we deal with in sociology are rarely independent of one another. The reciprocal interaction models stand in the middle in that they can treat interdependence on a smaller but understandable level.

Certainly the value of the notion of feedback is well known to sociological researchers. Many areas of social research, such as communications, decision making, social control, cannot be meaningfully analyzed without accounting for feedback in some way. Our purpose in this paper has been to suggest a technique for dealing with feedback that has been relatively ignored by sociologists. Currently researchers usually deal with feedback by means of ANOVA, simulations and various other types of controlled experimentation.

Since path analysis has been shown to be a powerful technique, particularly in helping us to determine various cause and effect relationships, it seems essential that its methods of dealing with feedback should become more fully explicated and developed. Many social situations cannot be characterized in simple cause and effect terms unless the situation is viewed as being static. Many, if not most, social situations require that we understand the reciprocal causal effects of the variables under consideration. The notion of feedback in path analysis is one method by which we may assess reciprocal causation. We recognize that this technique will not eliminate all of the confusion that results from "causal analysis." However, it should assist us in a variety of situations encountered by sociologists. It is unrealistic to assume that social systems are closed and, thus, we must, in some manner, seriously deal with the notion of feedback if our discipline is to continue to develop.

FOOTNOTES

1. We will be concerned with only those assumptions that are unique to feedback models. Assumptions about sampling and measurement are almost the same as those necessary for the unidirectional path analysis. See Heise (1969) for these assumptions.

REFERENCES

Black, C. E.
1967 The Dynamics of Modernization. New York: Harper and Row.

Blalock, H. M., Jr.
1964 Causal Inferences in Nonexperimental Research. Chapel Hill: The University of North Carolina Press.

1967 "Path coefficient versus regression coefficient." American Journal of Sociology. 71: 37-47.

Bogue, D. J.
1969 Principles of Demography. New York: John Wiley.

Boudon, R.
1965 "A method of linear causal analysis: dependence analysis." American Sociological Review 30: 365-72.

Duncan, O. D.
1966 "Path analysis: sociological examples." American Journal of Sociology 72: 1-16.

1970 "Duncan's correction of published text of 'peer influence on aspirations: a reinterpretation." American Journal of Sociology 75: 1042-46.

Duncan, O. D., Haller, A. O. and Portes, A.
1968 "Peer influence on aspirations: a reinterpretation." American Journal of Sociology 75: 119-37.

Ezekiel, M. and Fox, K. A.
1959 Methods of Correlation and Regression Analysis. New York: Wiley, 2nd ed.

Goldberger, A.
1964 Econometric Theory. New York: Wiley.

Heise, D. R.
1969 "Problems in path analysis and causal inferences." Pp. 38-73 in E. F. Borgatta (ed.) Sociological Methodology 1969. San Francisco: Jossey-Bass.

Johnston, J.
1963 Econometric Methods. New York: McGraw-Hill.

Land, K. C.
1969 "Principles of path analysis." Pp. 1-37 in E. F. Borgatta (ed.) Sociological Methodology 1969. San Francisco: Jossey-Bass.

Lerner, D.
1958 The Passing of Traditional Society: Modernizing the Middle East. New York: The Free Press.

Moore, W.
1963 Social Change. Englewood Cliffs: Prentice-Hall.

Roger, E. M.
1969 Modernization Among Peasants: The Impact of Communication. New York: Holt, Rinehart and Winston.

Tukey, J. W.
1954 "Causation, regression and path analysis." Pp. 35-66 in O. Kempthorne and others (eds.) Statistics and Mathematics in Biology. Ames, Iowa: Iowa State College Press.

Turner, M. E. and Stevens, C. D.
1959 "The regression analysis of causal paths." Biometrics, 15: 236-258.

Wold, H. and Jureen, L.
1953 Demand Analysis: A Study in Econometrics. New York: Wiley.

Wright, S.
1921 "Correlation and causation." Journal of Agricultural Research 20: 557-85.

1934 "The method of path coefficient." Annals of Mathematical Statistics 5: 161-215.

1960a "Path coefficient and path regression: alternative or complimentary concepts?" Biometrics 16: 189-202.

1960b "The treatment of reciprocal interaction with or without lag, in path analysis." Biometrics 16: 423-45.